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NONLINEAR OSCILLATIONS OF A GAS IN A CLOSED TUBE

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Introduction. Longitudinal oscillations of a gas in a tube, one end of which is stationary and the other moves in accordance with a given periodic law, are discussed. The tube walls are assumed to be perfectly rigid. Small oscillation amplitudes are dealt with in many experimental studies and analytic solutions [1-7]. They can therefore be used to test the numerical approaches employed. However, oscillation regimes in a tube at high amplitudes of piston displacements have not been studied thus far. This is chiefly due to the limitations of the method of investigation employed, which usually are based on perturbation methods.

In the present work, the study of wave processes is based on a numerical integration of the equations of gas dynamics [8, 9]; this made it possible to examine transient processes from the initial state to a periodically recurring regime and to remove stringent restrictions on the oscillation amplitude.

We are dealing with oscillations involving a change in excitation frequency ω near half the fundamental frequency of the tube $\Omega = \pi c_0/L$, where c_0 is the sound velocity in the unperturbed medium at the initial instant $t = 0$, and L is the tube length (the distance between the closed end and the mean position of the piston). We are dealing with the dependence of the solution on the ratio of half the path of the piston l to the tube length L over a wide range of l/L. Viscous effects in the gas are not considered, this being permissible for tubes of relatively large diameter [5].

Statement of the Problem. The numerical integration of the equations of ideal-gas dynamics is carried out in the two-dimensional region $0 \le x \le R$, $y_p(t) \le y \le L$ (Fig. 1), provided that the line $x = 0$ is the axis of symmetry, and the normal components of the velocity vector at the boundaries $y = L$ and $x = R$ are zero. The position of the left-hand boundary $y_p(t)$ is defined by the equation

$$y_p(t) = y_0 + \int_0^t v_p(t)dt,$$

where $y_0 = y_p(0)$, and $v_p(t)$ is given by the equations

$$v_p = l\omega\sin\varphi; \tag{1}$$

$$v_p = l\omega\left[\sin\varphi - \frac{b}{2}\frac{\sin 2\varphi}{\sqrt{1 - b^2\sin^2\varphi}}\right]. \tag{2}$$

Here $\varphi = \omega t + \varphi_0$; $\varphi_0 = \varphi(0)$; $b = l/a$; a is the length of the connecting rod. At the initial instant $t = 0$, the parameters of the problem were taken as follows: $\varphi_0 = \pi/2$, $y_0 = 0$, density $\rho(x, y) = \rho_0$, specific internal energy $I(x, y) = I_0$, velocity $v(x, y) = v_0 = 0$ everywhere except $y = y_0$, where $v(x, y_0) = v_p(0)$.

In addition, $b = 0.2732$, which corresponds to the experiment in [3]. In all known theoretical papers, the law (1) was used. The expression (2) is the law of oscillation of a piston in a motor and was discussed only in [8, 9].

The calculation was carried out using the two-dimensional method in dimensionless variables. The parameters used in the nondimensionlization were unperturbed values of density ρ_0 and sound velocity c_0, and the tube length L. In the calculations, the size of the cells along the Oy axis was determined by the equality $\Delta y = (L - y_p(t))/N_y$ (N_y being the number of grid cells along the Oy axis). Along the radial coordinate in the grid, there was only one element $\Delta x = R = 0.1L$. Thus, the grid was mobile in the longitudinal direction and had a constant radius.

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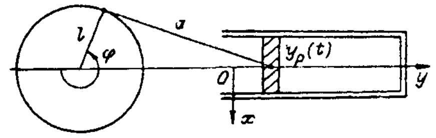

Fig. 1

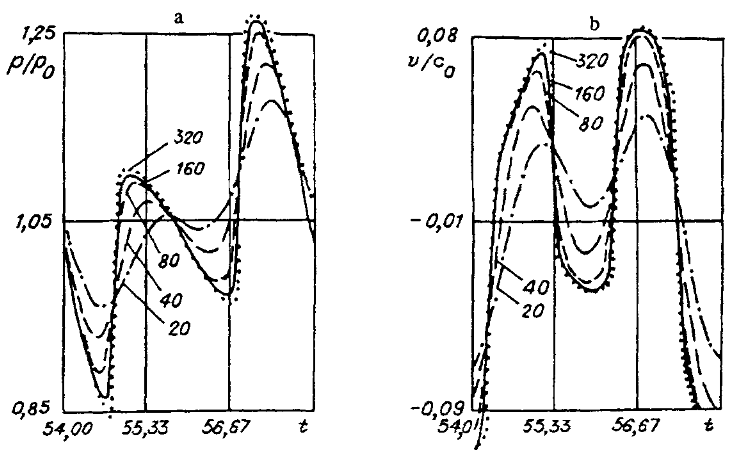

Fig. 2

Selection of the number of elements and numerical convergence. Figure 2 presents results of calculations on different grids: N_y = 20, 40, 80, 160, 320. Shown is the change in pressure with time at a point near the closed end of the tube (a) and the change in velocity at a point located at an equal distance from the piston and from the stationary end of the tube (b). The input data of the problem are: $\rho_0 = 1.03235$, $I_0 = 1.8085$, $l/L = 0.03225$, $b = 0.25$, $\varphi_0 = 90°$, $\omega = 0.5\Omega$, $y_p(t = 0) = 0$, law of piston oscillation (2). The fundamental characteristics of the solution with all the extreme values of pressure are conveyed in all the calculation variants. In addition, the numerical values are similar only when $N_y \geq 80$.

On the basis of calculations made on different grids, it may be concluded that in order to analyze problem in the category under consideration, it is necessary to carry out calculations using no fewer than 80 cells. This statement applies to the change in pressure. However, where necessary, the number of cells may be increased.

A number of calculations were made to estimate the contribution of the radicand and the denominator of Eq. (2) to the solution. It was found that the presence of a radical has no significant effect. In Fig. 3, which shows the time dependences of the pressure at the closed end, the results coincide graphically for the laws $v_p = -0.5l\omega b \sin 2\varphi$ and $v_p = -0.5l\omega b \sin 2\varphi/\sqrt{1 - b^2 \sin^2\varphi}$. The solutions are similar both for small values of l/L ($l/L = 0.03$) and for large ones ($l/L = 0.15$).

Dependence of the Solution on the Form of Excitation. Figures 4 and 5 show graphs of the time dependences of the pressure near the closed end of the tube. The dashed curves are the calculated ones. Figure 4 pertains to law (1), and Fig. 5 to law (2). Figure 6 shows the double amplitude of oscillations during a period at the same point as a function of oscillation frequency near $\omega = 0.5\Omega$, and the dashed lines also are the calculated ones here; among them, curves 2 and 3 pertain to laws (1) and (2), respectively, and 2 corresponds to excitation, which is obtained from Eq. (2) by neglecting the first term in square brackets. From Figs. 4-6, where $l/L = 0.03225$, it is evident that the transition from law (2) to law (1) is accompanied by significant changes. Thus, the double amplitude of the pressure oscillations at $\omega = 0.5\Omega$ during a period is reduced by a factor of almost 2. The oscillation shape itself also changes: in the case of law (1) (Fig. 4), there occurs an alternation in magnitude of the jumps, which differ by a factor of approximately 2. In the case of law (2), the jumps remain almost unchanged.

For comparison, the continuous lines in Fig. 6 represent curves obtained from the equation

$$p = -\frac{\rho_0 c_0 \omega^* l^*}{\sin(\omega^* L/c_0)} \cos\omega^* t,$$

which is the solution of equations of linear acoustics with the boundary conditions

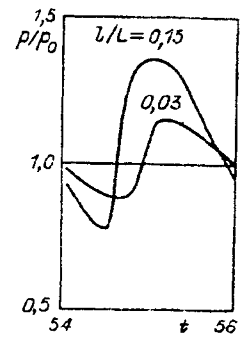

Fig. 3

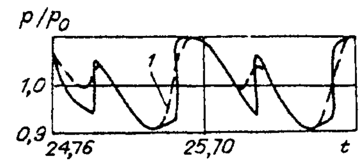

Fig. 4

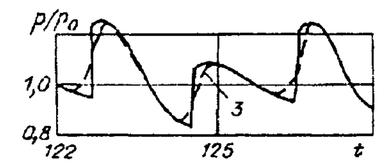

Fig. 5

$$v_p = \omega^* l^* \sin\omega^* t. \tag{3}$$

Curve 1 is obtained from (3) when $\omega^* = \omega$, $l^* = l$, and curve 2 is obtained when $\omega^* = 2\omega$, $l^* = -lb/4$. Clearly, in the first case $\omega = 0.5\Omega$ will give the first nonlinear resonance for the law (3), and in the second case, the first linear resonance.

Oscillations during High Excitation. It is well known that at small oscillation amplitudes of the piston, outside the vicinity of resonances, the nonlinear terms are weakly manifested. Therefore, the solutions of linear and nonlinear problems should be similar. This is indeed observed in Fig. 6: as the deviation from $\omega = 0.5\Omega$ increases, dashed curves 1 and 2 converge to the continuous curves.

The effect of the nonlinear terms in the vicinity of the first nonlinear resonance at low values of the piston stroke is discussed in detail in [3], both theoretically on the basis of the solution of nonlinear equations of gas dynamics for constant entropy and experimentally. The analytic solution and experimental data from [3] are shown by continuous lines in Figs. 4 and 5, respectively. In both cases, good agreement is observed between them and the numerical solution. A discrepancy takes place only in the vicinity of discontinuities, this being entirely natural for numerical methods, which possess artificial shock-capturing viscosity.

Thus, a comparison with solutions of the linear and nonlinear problems as well as with experimental data for a short piston stroke shows that the numerical results correctly reflect the fundamental characteristics of the solution.

It is apparent from Fig. 6 that the maximum values of the dashed curves do not exactly correspond to $\omega = 0.5\Omega$ (a deviation of about 2% is observed). The same can be seen in Figs. 7 and 8. Figure 7 shows curves of double amplitude of pressure oscillations at the closed end of the tube during a time of $58 \le t \le 62$. The continuous lines correspond to frequency $\omega = 0.5\Omega$, and the dashed lines correspond to those frequencies in the vicinity of 0.5Ω at which the oscillation double amplitude takes on an extreme value. It is evident from Fig. 6 that as l/L increases, the discrepancy between the continuous and dashed curves also increases. When $l/L = 0.12$, it is $\approx$ 25% for curves 2.

The content and notation used in Fig. 8 correspond to Fig. 6. The difference is that here $l/L = 0.12$. A comparison of Figs. 6 and 8 shows that as the length of the piston stroke increases, a change takes place in both the behavior of the curves for the same laws of excitation and in the differences between the curves corresponding to different laws. For example, the deviation of the maximum value toward high frequencies along the abscissa increases for all the laws. However, the maximum values themselves and the degree of deviation for different laws are different. Thus, whereas for $l/L = 0.03225$ the maximum of curve 2 along the ordinate is located between the corresponding values of curves 1 and 3 (at almost the same distance from them), for $l/L = 0.12$ the mean position occupies the maximum of curve 1. In addition, here the difference between this maximum and the highest value of curve 2 appreciably exceeds (almost by a factor of 4) the analogous difference between the maxima of curves 1 and 3.

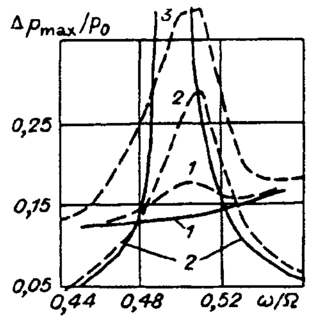

Fig. 6

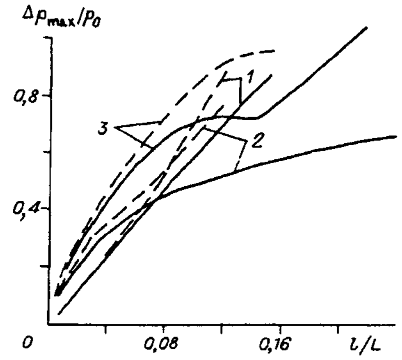

Fig. 7

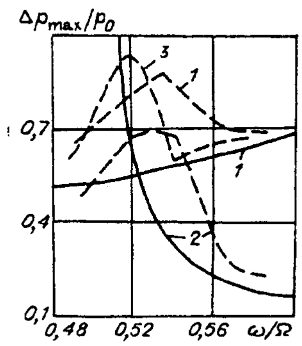

Fig. 8

An increase in l/L appreciably affects not only the magnitude of the oscillation double amplitude but also the shape of the oscillations. Figure 9 shows the change in pressure during a period at a point of the closed end of the tube in the range of l/L from 0.005 to 0.15 for the law (2) and frequency $\omega = 0.5\Omega$. It is evident that when $l/L = 0.005$, the deviations from the mean value $p/p_0 = 1.0$ to the higher and lower sides are almost the same. When $l/L = 0.15$, they already differ by a factor of more than 2.5, and as l/L increases, the front maximum steadily increases while the minimum reaches ≈ 0.75 when $l/L = 0.09$, then begins to increase again. Thus, as l/L increases, descent from resonance takes place. This is evident from both the change in oscillation shape (Fig. 9) and the magnitude of their double amplitude (Fig. 7).

Transient Regime. Use of the numerical method makes it possible to discuss transient processes in the entire range from the initial unperturbed state to a periodically recurring regime. For different frequencies, magnitudes of the piston stroke and laws of excitation, they differ in duration and in the character of change in parameters. For $l/L = 0.3225$, Fig. 10 shows the time curves of the pressure for $\omega = 0.51\Omega$, law (2) (Fig. 10), and for $\omega = 0.5\Omega$, law (1) (Fig. 11). It is evident that in the first case, the final oscillation shape is obtained almost immediately. As time goes on, there is only some change in the numerical values. The process becomes stabilized as early as $t \approx 16$. In the second case, the oscillations do not become stabilized until $t \geq 45$, and prior to that, appreciable changes take place in the shape of the oscillations, the amplitude of which initially decreases (to $t \approx 30$), then gradually increases to the steady-state values.

Conclusions. The studies show that for a short piston stroke ($l/L = 0.3225$), the results obtained on the basis of the numerical solution of the equations of gas dynamics are in good agreement with the solution of linear equations outside the resonance region as well as with the solution of nonlinear equations for the first nonlinear resonance $\omega = 0.5\Omega$. In that case, good agreement with experimental data is also observed. It was found that the presence of a radical in the denominator of the second law of excitation has little effect on the solution.

In addition, it was found that when $\omega = 0.5\Omega$ and l/L ranges from 0 to 0.20 in the initial portion of time, the law (2) being used, the first term, which represents the law (1) predominates in all the variants. Then the role of the second term increases, and become predominant for all values of time. When the piston moves ($l/L \geq 0.03$), the two terms make approximately the same contribution to the solution, and the effect of the first term increases more and more as l/L increases. Under the law (1), in the initial time interval, an appreciable rearrangement of oscillation shape takes from the initial form to the steady state.

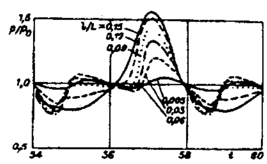

Fig. 9

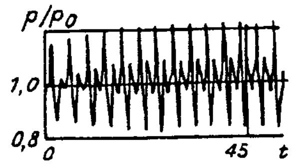

Fig. 10

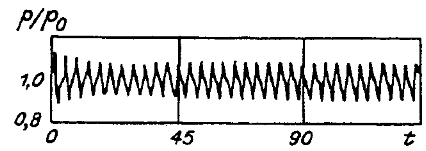

Fig. 11

On the whole, at high values of the piston stroke, the contribution of nonlinear effects increases appreciably, and as a result, the change in parameters under the general law (2) differs significantly from their change when its individual terms are used and from the simple sum of the results thus obtained.

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